What is risk treatment?

by Eric Marsden <eric.marsden@risk-engineering.org>

1 Risk treatment

Definition

Risk treatment (ISO 73 standard)

Risk treatment is the process of selection and implementation of measures to reduce risk.

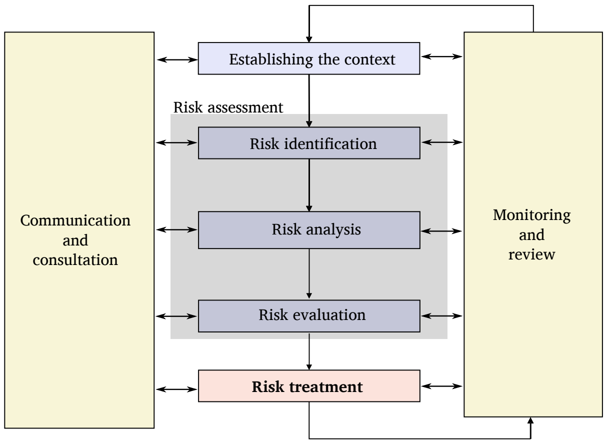


Figure 1 – *The contribution of risk assessment to the risk management process (adapted from the ISO 73 standard)*

Risk treatment methods:

- risk avoidance: cease the hazardous activity and reduce probability of loss to zero (but also lose the benefits of the activity!)
- risk modification
  - by reduction or containment (prevention, before event) (*safety relief value*)
  - by mitigation (protection, post-event) (*bunds, fire alarms*)
- risk sharing or transfer
  - diversification (*sell both heating and air-condition equipment*)
  - hedging (*farmer purchases protection against prices of wheat decreasing*)
  - insurance (*credit default swap*)

2 How much risk should I transfer?

Definition

Expected value

The expected value of a gamble is the value of each possible outcome times the probability of that outcome. It is the amount that I would earn of average if the gamble were repeated many times. For a binary choice between A and B, $\mathbb{E}(W) = Pr(A) \times W_A + (1 - Pr(A)) \times W_B$, where W_A is my wealth if outcome A occurs.

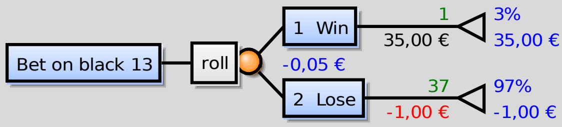
Example

Playing black 13 in roulette

The expected value of betting 1€ on black 13 in American roulette (which has 38 pockets numbered 1 to 36 plus 0 plus 00, and a payout for a single winning number of 35 to one[1]) is

$$35€ \times \frac{1}{38} + -1€ \times \frac{37}{38} = -0.0526€$$

This means that each time you place a bet in the roulette table, you should expect to lose 5.26%. The figure below shows a *decision tree* for this situation.



Similar types of graphical representations can be used to help decide whether a particular safety investment is worthwhile, given the probabilities of failure with or without the investment, and the estimated consequences in case of failure.

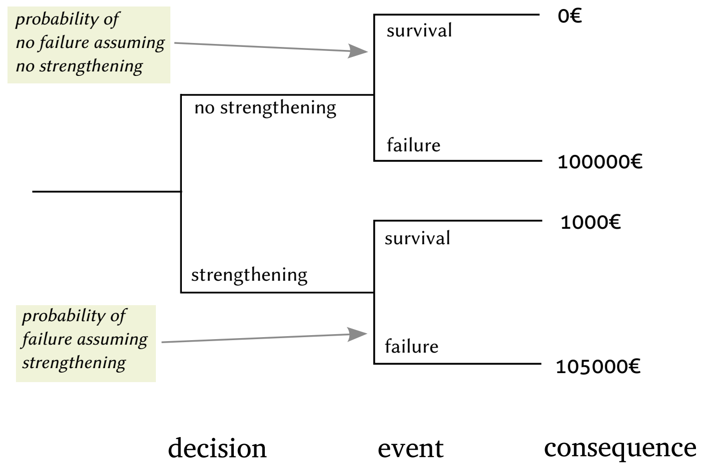


Figure 2 – *Example of a decision-event tree, showing estimated loss with or without safety investment*

3 Expected utility

Example

The Saint Petersberg paradox

In the “Saint Petersberg game”, you flip a coin repeatedly until a tail appears. The pot starts at 1€ and doubles every time a head appears. You win whatever is in the pot the first time you throw tails, and the game ends. For example:

- tail on the first toss: win 1€
- tail on the second toss: win 2€
- tail on the third toss: win 4€
- tail on the nth toss: win 2^{n-1}€

The expected value of this game is infinity. Indeed, with probability 1/2 you win 1€, with probability 1/4 you win 2€, probability 1/8 you win 4€, probability 1/16 you win 8€, and so on (without any limit, since in theory you can continue throwing heads indefinitely). Thus the expected value of your winnings is

$$\mathbb{E}(W) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots = \infty$$

[1] The initial bet is returned as well as 35€ for each euro bet.

The St Petersberg paradox indicates that people do not use expected value to make decisions concerning gambles. Bernoulli (1738) suggested that the "value" of a gamble is not its monetary value, but that people attach some subjective value, or *utility*, to monetary outcomes. People do not seek to maximize expected value, but instead maximize *expected utility*. Marginal utility decreases a wealth increases (poor people value increments in wealth more than rich people do). This is another way of saying that people are "risk averse".

A utility function $U(x)$ is a measure of goal attainment or want satisfaction for good x. Utility functions are monotonically increasing: more is always preferred to less (which means that $U'(x) > 0$). The *marginal utility* of x is the change in utility resulting from a small change in x (it's the slope of the utility function). utilité marginale

$$MU(x) \stackrel{\text{def}}{=} \frac{\Delta U(x)}{\Delta x}$$

The principle of **diminishing marginal utility** means that each successive unit of a good yields less utility than the one before it.

The expected utility of a gamble is the probability-weighted average of the utility from the potential monetary outcomes. For a binary choice between A and B,

$$EU \stackrel{\text{def}}{=} Pr(A) \times U(W_A) + Pr(B) \times U(W_B)$$

In classical economics, the **utility function** U is a way of modelling people's behaviour when faced with risk[2].

4 Risk aversion

Risk aversion (in psychology and economics) is the reluctance of a person to accept a gamble with an uncertain payoff rather than another bargain with a more certain, but possibly lower, expected payoff. For example, a risk-averse investor might choose to put his or her money into a bank account with a low but guaranteed interest rate, rather than into a stock that may have high expected returns, but also involves a chance of losing value.

Example

Risk aversion

I have 10€. Suppose I can play a gamble with 50% chance of winning 5€, and 50% chance of losing 5€.

The expected value of my wealth if I play the gamble is 10€ (the same as if I don't play!). The expected utility is $0.5U(15€) + 0.5U(5€)$. If I am risk averse, the utility of the expected wealth $U(10€)$ is bigger than the expected utility of wealth ($0.5U(5€) + 0.5U(15€)$). My utility function is therefore *concave*.

The **certainty equivalent value** of a gamble is the sum of money for which an individual would be indifferent between receiving that sum and taking the gamble. If I am risk averse, the certainty equivalent value is less than the expected value (I don't like taking risks, so you need to pay me for me to accept). The **risk premium** is the difference between the expected value and the certainty equivalent value. This is the "cost of risk": the amount of money that an individual would be willing to pay to avoid risk. It's also the value of **insurance**. prime de risque

Insurance companies work by pooling together many people's risks. Each person taking the insurance pays a bit more than the "real" (or mathematically fair) value of their risk. Since it has a large number of clients, the insurance company can play the "large numbers" game many times, and will overall probably win money. The larger the insurance company, the better the law of large numbers works.

[2] In economics, *risk* designates situations where the future state is unknown but the probability of each possibility is well known, and *uncertainty* designates situations where the probability of future states is not well known. The term *radical uncertainty* is used when the possible future states are not clearly delimited.

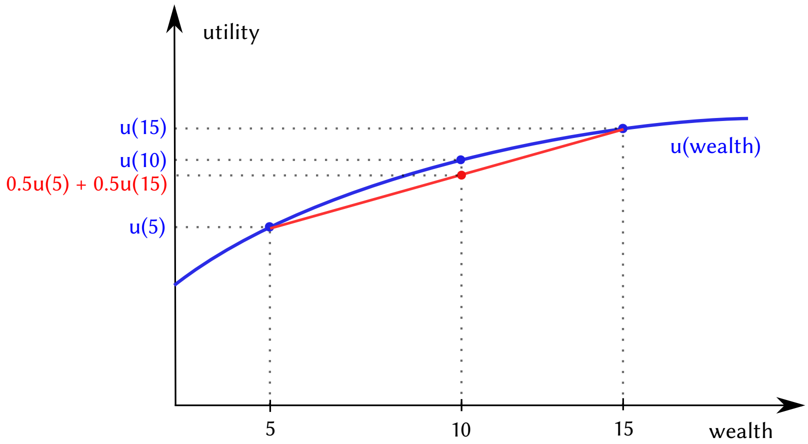


Figure 3 – *A risk-averse person has a concave utility function*

Example

Willingness to pay for insurance

Consider someone with a current wealth of 100k€, who faces a 25% chance each year of losing his automobile, worth 20k€. The expected value of his loss is $0.25 \times 20\,k€ = 5\,k€$, so the expected value of his wealth at the end of the year is 95k€.

We will assume that this person has *log utility* (his utility function is $U(x) = log(x)$). The person's expected utility will be

$$\begin{aligned} EU &= 0.75U(100k) + 0.25U(80k) \\ &= 0.75log(100k) + 0.25log(80k) = 11.45 \end{aligned}$$

This (risk-averse) person will likely be willing to pay more than the expected value of the loss (5 k€) to avoid the risk. How much will he pay for insurance? Let's calculate the cost of insurance (y) that will leave him indifferent compared with not having insurance:

$$\begin{aligned} EU = U(100k - y) = log(100k - y) &= 11.45 \\ 100k - y &= e^{11.45} \\ y &= 5426 \end{aligned}$$

The maximum amount this person would be willing to pay for insurance is 5426€, so his risk premium (the expected profit for the insurance company) is 426€.

5 Stopping risk treatment: judgment of acceptability

The classical way of evaluating risk is to position an accidental scenario in a **risk matrix**, which in one dimension contains classes of magnitude (severity of impact) of the possible adverse consequences, and in the other dimension classes of likelihood (probability) of occurrence of each consequence. Each organization will decide on the thresholds between the acceptable, ALARP and unacceptable regions in its risk matrix, and generally regulators have their own risk matrix which they will use to assess an Operator's safety case.

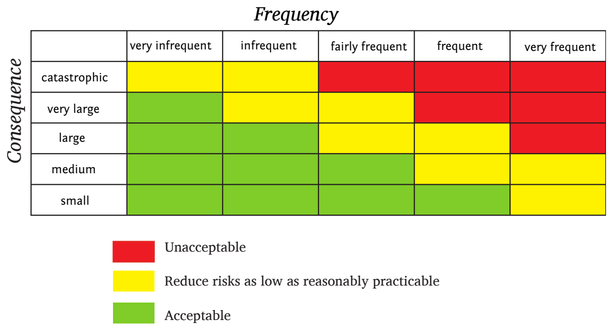


Figure 4 – *A typical risk matrix, showing the acceptable (green), ALARP (yellow) and unacceptable (red) regions*

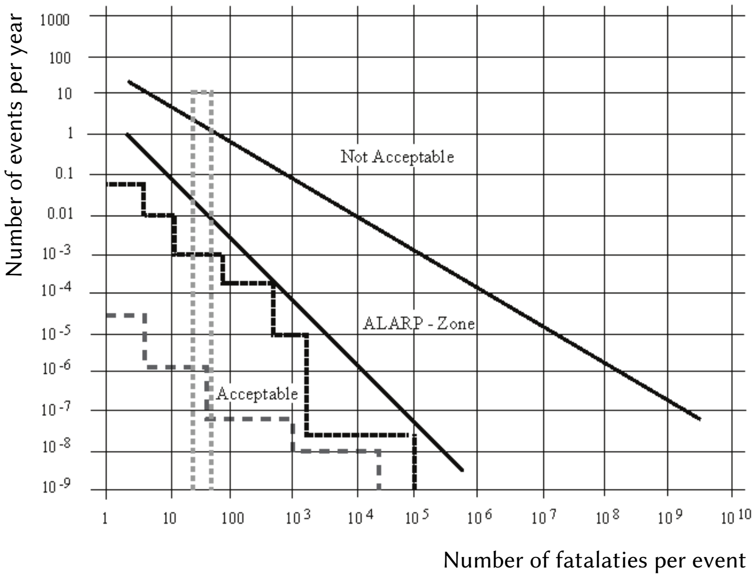


Figure 5 – *Farmer (or F-N) diagram showing acceptable risk region, ALARP zone and non-acceptable region*

This document is part of the risk-engineering.org course materials. It is distributed under the Creative Commons Attribution-ShareAlike licence: you can share and reuse the content as long as you attribute it to the author and redistribute it under the same terms.