Black swans,
or the limits of statistical modelling

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Do I follow indicators that could tell me when my models may be wrong?
A turkey is fed for 1000 days.

Each passing day confirms to its statistics department that the human race cares about its welfare “with increased statistical significance”.

On the 1001st day, a little before Thanksgiving, the turkey has a surprise.
A turkey is fed for 1000 days.

Each passing day confirms to its statistics department that the human race cares about its welfare “with increased statistical significance”.

On the 1001st day, a little before Thanksgiving, the turkey has a surprise.
“Things that have never happened before, happen all the time.


In risk analysis, these are called “unexampled events” or “outliers” or “black swans”.
The term “black swan” was used in 16th century discussions of impossibility (all swans known to Europeans were white).

Explorers arriving in Australia discovered a species of swan that is black.

The term is now used to refer to events that occur though they had been thought to be impossible.
Characteristics of a black swan event:

- an outlier
  - lies outside the realm of regular expectations
  - nothing in the past can convincingly point to its possibility
- carries an extreme impact

Note that:

- in spite of its outlier status, it is often easy to produce an explanation for the event after the fact
- a black swan event may be a surprise for some, but not for others; it’s a subjective, knowledge-dependent notion
- warnings about the event may have been ignored because of strong personal and organizational resistance to changing beliefs and procedures

September 2001 terrorist attacks in New York
“That was a risk factor for avian flu that we hadn’t really considered before.”

– Epidemiologist Tim Uyeki (US CDC), after a man became infected by sucking a rooster’s beak to clear its airways, a technique not uncommon in cockfighting circles.

Europeans “discovering” America was a black swan event to the native population (of which 80 to 90% died after arrival of Europeans)
Financial collapses

Daily variations of a derivatives portfolio exposed to UK interest rates between 1988 and 2008.

≈ 99% of the variability comes from a single day (collapse of the European Monetary System in 1992).

This one event is critical to the risk level, but it is very difficult to estimate its likelihood with standard statistical/econometric techniques.

Figure source: The fourth quadrant: a map of the limits of statistics, Nassim Nicholas Taleb (2007)
Rare events are not all black swans

▷ The Fukushima Daiichi tsunami (March 2011, 14 m wave) was caused by a magnitude 9 earthquake

▷ There were two recorded events of similar magnitude in that area (Sanriku coast):
  • 9th century: earthquake of estimated magnitude 8.6
  • 17th century: earthquake of estimated magnitude 8.1, 20 m tsunami

▷ These historical records seem not to have been taken into account when designing the Fukushima Daiichi nuclear power plant
  • design basis was a maximum wave height of 5.7 m

▷ The 2011 tsunami could have been imagined using available historical information
  • not a black swan according to our definition
Risks of extrapolation

Source: xkcd.com/605/, CC BY-NC licence
Some fundamental assumptions in risk management:
• You can identify hazards in a relatively exhaustive manner
  • the set of possible future events is delimited and known
  • no “unknown unknowns”
• You can estimate the probability of future events and the magnitude of their consequences
• If the risk is not acceptable to you, you can act to modify the probabilities or the magnitude of consequences

Many of these assumptions are invalid when dealing with black swans and situations with high uncertainty
Statistical characteristics of black swans
“Thin-tailed” probability distributions

▷ The “tail” of a probability distribution is the part which is far away from the mean

▷ Typical examples of thin-tailed probability distributions:
  • normal (Gaussian) distribution
  • exponential distribution

▷ For a normal distribution:
  • probability of an observation which is more than 6 standard deviations from the mean (a “six-\(\sigma\) event”) is \(9.9 \cdot 10^{-10}\)
  • a 10-\(\sigma\) event is less frequent than one day in the life of the universe
“Fat-tailed” probability distributions

- Extreme events carry significant weight
  - fairly high probability of something “unusual” occurring

- More formally, the tail of the distribution follows a power law:

  \[ \Pr[X > x] \sim x^{-\alpha} \text{ as } x \to \infty, \quad \alpha > 0 \]

  where \( \sim \) means “is asymptotically similar to”.

- Example: the Pareto distribution
  - designed by economist V. Pareto to model distribution of wealth in capitalist societies
  - a good fit for the severity of large losses for worker compensation, the level of damage from forest fires, the energy released by earthquakes...

- Example: the oil shock in 1973 was a 37-sigma event
Two caricatures of probability distributions

**Mediocristan**
Events are easy to predict.

Exceptions occur, but don’t carry large consequences.

Example: distribution of population’s weights follows a Gaussian distribution. Add the heaviest person on the planet to a sample of 1000 people, and the total weight will barely change.

“thin tails”

**Extremistan**
Exceptions can be everything.

Example: distribution of wealth in capitalist economies. Add Bill Gates to your sample of 30 random people and mean wealth will jump by a factor of 100,000.

Winner takes all effects (sales of novels, albums).

Future is hard to predict from past information.

“fat tails”

Source: *The Black Swan: The Impact of the Highly Improbable* by N. Taleb
Winner takes all effects

- Less than 0.25% of all the companies listed in the world represent around half the market capitalization

- Less than 0.2% of books account for approximately half their sales

- The top 1% of bands and solo artists now earn 77% of all revenue from recorded music

- Less than 0.1% of drugs generate a little more than half the pharmaceutical industry’s sales

- “The rich get richer”

- Leads more often to “golden goose” events (extreme upside) than to “black swan” events (extreme downside)
Example: income inequality in USA

Change in Share of Total Income, 1967-2012
relative to 1967, by percentile

SOURCE: CENSUS BUREAU

Source: motherjones.com/politics/2013/09/charts-income-inequality-middle-class-census
“The rich get richer”

Data for USA, but analysis by economist T. Piketty suggests similar trends for most developed countries.

Greater income inequality means a more fat-tailed income distribution and more outliers in the population.

Source: motherjones.com/politics/2011/02/income-inequality-in-america-chart-graph
The Barabási–Albert model is an algorithm for generating random scale-free networks using a preferential attachment mechanism.

Generates a “fat-tailed” distribution following a Pareto distribution or power law in its tail.

Mechanism which might explain:
- distribution of the sizes of cities
- wealth of extremely wealthy individuals
- number of citations received by scientific publications
- number of links to web pages
Event probability in Mediocristan

Events in Mediocristan:

▷ In a developed country a newborn female is expected to die at around 79, according to insurance tables.

▷ When she reaches her 79th birthday, her life expectancy, assuming that she is in typical health, is another 10 years.

▷ At the age of 90, she should have another 4.7 years to go.

▷ If you are told that a person is older than 100, you can estimate that he is 102.5 and conditional on the person being older than 140 you can estimate that he is 140 plus a few minutes.

▷ The conditional expectation of additional life drops as a person gets older.
Event probability in Extremistan

- Events in Extremistan:
  - the conditional expectation of an increase in a random variable does not drop as the variable gets larger
  - conditional on a loss being worse than 5 units, it will be around 8 units
  - conditional on a move being more than 50 units, it will be around 80 units
  - if we go all the way until the sample is depleted, the average move worse than 100 units is 250 units!
  - there is “no typical” failure and “no typical” success

- Examples:
  - movements on the stock market
  - predictions of drug sales: sales estimates are often very different from actual sales
Coping with black swans
Attempting to cope with black swans

- By definition, black swan events are not predictable

- You can try to determine your level of exposure to them
  - is my system simple/complicated/complex/chaotic?
  - what is my level of confidence in our knowledge of the system?

- Coping techniques that try to **reduce their likelihood**:
  - training people to increase their “safety imagination”
  - stress tests
  - horizon scanning exercises: a methodical way of identifying threats and opportunities

- Coping techniques that try to **reduce their impact**:
  - logics of precaution
  - post-normal science and participatory governance
  - increased system resilience
Cynefin framework and complexity

- Describes five system contexts depending on relationship between cause and effect
- Contexts: obvious, complicated, complex, chaotic, disorder
- A “cliff” between obvious and chaotic
- Describes the types of issues that decision-makers face in these contexts
## Decision-making & black swans

<table>
<thead>
<tr>
<th></th>
<th>Simple payoffs</th>
<th>Complex payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin-tailed distributions</td>
<td>Very robust to black swans</td>
<td>Quite robust to black swans</td>
</tr>
<tr>
<td>Fat-tailed distributions or unknown tails</td>
<td>Quite robust to black swans</td>
<td>Susceptible to black swans (limit of statistics)</td>
</tr>
</tbody>
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Simple payoffs: binary outcomes

Complex payoffs: dependent on expected value, possibly leveraged

Source: N. Taleb
Safety imagination

“The only surprise is that we’re surprised when we’re surprised.”
— Former US Secretary of Defense Donald Rumsfeld

Safety imagination: concept proposed by N. Pidgeon to characterize ability to “think outside the box”

- help develop richer accident scenarios than those normally considered in risk assessments
- reduce complacency on safety issues
- can trigger discussion on new approaches to managing risks and emerging hazards
- conducive to learning
Fostering safety imagination: suggestions

▷ Attempt to fear the worst; allow no worst case situation to go unmentioned

▷ Use good meeting management techniques to elicit varied viewpoints

▷ Play the “what if” game with potential hazards
  • brainstorming to avoid groupthink

▷ Suspend assumptions about how the safety task was completed in the past

▷ Tolerance of ambiguity will be required when approaching the edge of a safety issue
  • newly emerging safety issues will rarely be clear

▷ Force yourself to visualize “near-miss” situations developing into accidents

Source: Man-made disasters: why technology and organizations (sometimes) fail, N. Pidgeon & M. O’Leary, Safety Science, 2000, DOI: 10.1016/S0925-7535(00)00004-7
FACTORS CONTRIBUTING TO UNKNOWN UNKNOWNS

The six factors shown here tend to increase the likelihood of unk-unks (surprises) in projects.

- equivocality
- dynamism
- complicatedness
- complexity
- mindfulness
- project pathologies

Social conformity is a strong yet subtle phenomenon

- The Asch conformity experiments in psychology analyzed (1950s) to what extent individuals yield to or defy a majority group & effect of such influences on beliefs

- Example experimental setup: a rigged “perceptual” task
  - a group of 8 participants are asked to say aloud, in turn, which line in the right-hand group matched the length of the left-hand line
  - all but one of the participants are working with the experimenter (“confederates”)
  - each confederate in turn gives the same incorrect response
  - the subject of the experiment is asked to answer last

- Measures normative social influence: willingness to conform publicly to attain social reward and avoid social punishment

Figure by Fred the Oyster via Wikimedia commons, GFDL
Social conformity is a strong yet subtle phenomenon

▷ Results: complex mixture of individual differences in subject’s reaction to the experimental situation
  • some individuals yield to group norm, some express doubt but conform, some resist (most frequent reaction)

▷ Striking example of people publicly endorsing the group response despite clearly knowing that they were approving an incorrect outcome
  • potentially dangerous feature of group dynamics!

▷ Related experiments:
  • Milgram (1960s) measured the willingness of people to obey an authority figure who instructed them to perform acts conflicting with their personal conscience (inflicting electric shocks on a “learner”)
  • Stanford prison experiment (1970s) assessed willingness of people role playing as prison guards to abuse their “prisoners”
Social conformity is a strong yet subtle phenomenon

- Famous experiment on the “bystander effect” or “diffusion of responsibility”
  - people defer to others on how to interpret ambiguous situations
  - don’t like overreacting to later appear ridiculous
  - if you observe other people doing nothing in response to an apparent risk, you’re more likely also not to respond

- Experiment: observe how people react to a room which progressively fills with smoke
  - first case: subject is along with the experimenter, 75% of them report the smoke
  - second case: subjects sitting in the room with passive people (confederates, told not to react by experimenter), only 10% report the smoke

A “red team”

- A red team is an independent group that provides an **adversarial perspective**, challenging an organization to improve its effectiveness
  - imagines attacks from a competitor, worst-case futures, and nightmare scenarios
  - challenges assumptions and examines how well current plans would work

- **Goal:**
  - to think through consequences of the bad event
  - to consider what measures might be possible now to improve ability to deal with the results
  - not to precisely predict the threats or to plan in detail

- Concept developed by the military

- Extension of the “devil’s advocate” concept in group discussions
Scenario analysis follows a systematic process to create a set of **plausible** and vividly **contrasting narratives** that describe possible **evolutions of key areas of uncertainty**

These narratives (scenarios) examine the social, political, economic, and technological forces that might affect a project or strategy.

They are developed by a **diverse team** including decision-makers, experts and stakeholder representatives.

Can be used to illuminate **“wild cards”** (high impact, low probability events that happen so fast that social systems cannot effectively respond)

- example: possibility of the earth being struck by a meteor

A method for developing **foresight** (thinking about possible futures) which feeds in to strategy development
TRIZ: theory of solving problems creatively

- The TRIZ method (theory of resolving invention-related tasks) was developed by Russian inventors

- Steps in a “reverse TRIZ” used for hazard analysis:
  1. Start with the objective
  2. Reverse this objective
  3. Exaggerate/amplify the objective (hazard)
  4. Identify resources needed to reach the objective

- Benefits of this approach:
  - puts participants in a positive mindset about identifying potential hazards (can reduce lack of engagement in the process)
  - encourages people to free their imagination

Source: Critical Uncertainties blog by Matthew Squair
Example application of TRIZ to a hazard analysis

▷ Objective: Minimise the risk of a plant gas leak and explosion

▷ Reversed: I want to increase the risk of a plant gas leak and explosion

▷ Amplified: I want lots of gas leaks and when they occur big explosions!

▷ Resources:
  • Lots of pipework joints
  • Store lots of gas in the system
  • Make sure that gas can leak into areas where it doesn’t disperse
  • Ensure that there’s lots of maintenance...
Scenario analysis

Diagram of the Scenario Process, as used by World Bank

Choose scenario group → Preliminary interviews → Preliminary research → First scenario → 4-5 draft scenarios → Verify scenario assumptions → Write full-length scenarios → Implement operation and monitor progress → Communicate scenarios → Indicators for external events in each scenario → Strategic implications of each scenario → Final scenario workshop(s) → 4-5 final scenarios → Review and revise scenarios → Final scenario workshop(s)

Legend:
Core scenario team
Full scenario team

Figure source: Scenario Analysis: A Tool for Task Managers, World Bank report
The difficulty of predicting

“Prediction is very difficult, especially if it’s about the future.”
— Nils Bohr (disputed quotation)

“Prophesy is a good line of business, but it is full of risks.”
— Mark Twain in *Following the Equator*
Heavier than air flying machines are impossible.

— Lord Kelvin, President of the Royal Society, 1895
I think there is a world market for maybe 5 computers.
— Thomas Watson, president of IBM Corporation, 1943
Who the hell wants to hear actors talk?

— H. M. Warner, Warner Bros, 1927
Anticipatory Failure Determination (AFD)

- AFD is an inventive problem solving technique

- Standard failure analysis methods such as FMEA: ask designer to look for a cause of a failure mode

- AFD reverses this approach, asking designers to
  - view failure of interest as the intended consequence
  - try to **imagine ways to ensure that the failure always happens repeatably**

- Can help to overcome the phenomenon of denial ("oh, that wouldn’t happen to our product")

- Can increase amount of creativity deployed in analysis of failures
Anticipatory Failure Determination: steps

1. **Inverse the formulation** of the problem: not “how did it happen?” but identifying steps to produce the problem
   - identify the conditions that lead to the failure
   - identify the scenario or events involved

2. Search for methods to **reproduce the failure**
   - identify all the standard ways of creating the failure
   - for example using function analysis methods

3. Verify that **resources** are available to cause the failures
   - resource categories: substances, field effects, space available, time, object structure, system functions, and other data on the system
   - for each of the potential solutions, identify whether the required resources are available to allow failure
Stress testing

- Stress test: an analysis or simulation designed to determine the ability to deal with an exceptional event
  - tests robustness with respect to catastrophes
  - a complement to standard “best estimate” projections and risk analyses

- Widely used in finance, with scenarios such as:
  - “What happens if the unemployment rate rises to $X\%$ in a specific year?”
  - “What happens if GDP falls by $X\%$ in a given year?”
  - “What happens if interest rates go up by at least $X\%$?”
  - “What happens if oil prices rise by 200%?”

- Note: a stress test is only as good as the scenarios on which it is based
Example applications of stress testing

▷ Used by regulators to ensure that banks and insurance companies have adequate capital levels to resist to extreme — but plausible — events
  • in particular since the 2008 financial crisis

▷ Regulatory stress testing in the banking sector to ensure capital levels are adequate
  • Comprehensive Capital Analysis and Review (CCAR) in the USA
  • ECB “Comprehensive Assessment” stress tests in the EU
  • avoid possibility of systemic risks in the financial sector (domino effects where failure of one organization leads to a cascade of failures)

▷ Also used by businesses, insurers and governments to analyze emerging threats and possible vulnerabilities
  • example: University of Cambridge Center for risk studies publishes threat scenarios at cambridgeriskframework.com
Aside: video on systemic risks in finance

View video online: youtu.be/UzW195qWHYg

More information: systemic-risk-hub.org
Example applications of stress testing

The insurer Lloyd’s maintains a set of mandatory Realistic Disaster Scenarios to stress test both individual syndicates and the market as a whole. The event scenarios are regularly reviewed to ensure they represent material catastrophe risks.

Source: lloyds.com/the-market/tools-and-resources/research/exposure-management/realistic-disaster-scenarios
Post-normal science

▶ A concept developed by Silvio Funtowicz and Jerome Ravetz for situations where “facts are uncertain, values in dispute, stakes high and decisions urgent”

▶ When using scientific knowledge for policy-making, some elements should be made explicit:
  - nature of uncertainty present in the analysis (stochastic, epistemic or axiomatic/values-based) and an estimate of level of uncertainty
  - the quality (or “pedigree”) of the analysis (are the data and assumptions new or have they had time to mature?)
  - the stakes of the decision (costs and benefits and value commitments of all stakeholders, impacted timeframe)
  - the impact of different value systems and ethics on the possible decisions

Source: Science for the post-normal age, S. Funtowicz & J. Ravetz, Futures, 1993
Post-normal science

▷ Related notion of *trans-science*: questions that can be asked of science, but not answered by it
  - risk problems which cannot be resolved by sound science or evidence-based risk assessment
  - questions of fact which can be stated in the language of science but not answered
  - questions that “transcend” science

▷ Example: biological effects of exposure to low-level ionizing radiation
  - testing whether genetic response to radiation is linear (including at very low doses) would require experiments on billions of mice
  - not feasible in practice
  - cannot be answered by direct scientific investigation

“\textquote{I would rather have questions that can't be answered than answers that can't be questioned.}”
\textquote*{Richard Feynman}

Source: *Science and trans-science*, Alvin Weinberg (1972), Minerva, 10, 1972, 209-222
Post-normal science

systems uncertainty

decision stakes

low

high

low

high
Post-normal science

- decision stakes
- applied science
- traditional scientific method

systems uncertainty
Post-normal science

decision stakes

low

high

systems uncertainty

low

high

applied science

professional consultancy

judgment and good practice
Post-normal science

- High decision stakes
- High systems uncertainty
  - Societal and community consensus and community review
- Low decision stakes
- Low systems uncertainty
  - Professional consultancy
  - Applied science
  - Post-normal science
Post-normal science

▷ **pure research** is motivated by curiosity

▷ **applied science** is “mission-oriented”
  • uses the traditional scientific method
  • affected by technical uncertainty, which is managed by statistical analysis

▷ **professional consultancy** is “client-serving”
  • uses professional judgment and good practice
  • affected by methodological uncertainty, which is managed by professional consensus and peer review

▷ **post-normal science** is “issue-driven”
  • affected by epistemological uncertainty, managed by societal and community consensus and community review
  • quality assurance for scientific information is an acute problem (*cf.* nusap.net)
Coping technique: precautionary logics

- Scenarios involving very large potential impacts and uncertainty concerning probability are poorly suited to classical decision-making
  - examples: climate change, GMOs, nuclear war

- These scenarios call for
  - logics of precaution, such as Jonas’ imperative of responsibility, the precautionary principle, sustainability principles
  - deliberation about action: techniques from participatory democracy and social appraisal can provide a complement to existing scientific knowledge concerning an uncertain risk

- Note: the continued failure of international initiatives such as the UN Convention on Climate Change and the UN Convention on Biological Diversity illustrate the difficulties of working together on complex, high-impact but uncertain challenges...
Coping technique: increase resilience

- Limits in knowledge mean we can’t determine the probability of black swan events

- However, we can act to limit their impact, by **reducing vulnerabilities**

- Notion of *system resilience*, after E. Hollnagel and D. Woods:
  - ability of a system to adjust its functioning prior to, during, or following events (changes, disturbances, and opportunities), and thereby sustain required operations under both expected and unexpected conditions

- Characteristics of resilient organizations: ability to
  - *respond* to variability, disturbances and opportunities
  - *monitor* what is going on and identify threats and interpret them
  - *anticipate* possible future outcomes and their potential
  - *learn* from experience
The four cornerstones of resilience

Ability to learn from the past

Knowing what to do

Responding (actual)

Learning (factual)

Knowing what happened

Monitoring (critical)

Knowing what to look for

Anticipating (potential)

Knowing what to expect

The four cornerstones of resilience

- **Learning** (factual)
  - Knowing what happened

- **Monitoring** (critical)
  - Knowing what to look for

- **Anticipating** (potential)
  - Knowing what to expect

- **Responding** (actual)
  - Knowing what to do when something happens, either by following previous practices or being able to adjust to the new situation quickly

The four cornerstones of resilience

Knowing what happened

Knowing what to do

Knowing what to look for

Knowing what to expect

Being able to see the early signs that something is changing and taking timely avoiding action

The four cornerstones of resilience

Knowing what happened

Learning (factual)

Knowing what to do

Responding (actual)

Knowing what to look for

Monitoring (critical)

Ability to anticipate what may happen in the future and what effect it will have on the organization

Knowing what to expect

Anticipating (potential)

Coping technique: increase resilience

▷ Lots of research on resilience engineering over the past 10 years
  • → resilience-engineering-association.org
  • see Eurocontrol White Paper on resilience engineering for ATM

▷ A related principle developed by the High Reliability Organizations (HRO) group of researchers is sensitivity to operations
  • everyone maintains an awareness of what is and isn’t working in the system
  • the opposite of complacency on safety matters
  • attention paid to weak signals or early warning signals

▷ Related principles from the HRO work which contribute to collective mindfulness:
  • reluctance to simplify interpretations
  • preoccupation with failure
  • deference to expertise
- Turkey (slide 2): flic.kr/p/beoUXF, CC BY-SA licence
- Black swan (slide 4): flic.kr/p/edfYvX, CC BY licence
- United Airlines Flight 175 crashes into the WTC south tower on 9/11 (slide 6): Robert J. Fisch via Wikimedia Commons, CC BY-SA licence
- Cock fight (slide 7): Rom via flic.kr/p/nQBsCM, CC BY-NC-ND licence
- Columbus discovering Guanahani, Bahamas (slide 8), engraving by T. de Bry, circa 1590, public domain
- Jelly beans (slide 13): Steve Petruelli via flic.kr/p/btR61D, CC BY-NC-ND licence
- Desert in Namibia (slide 21): jbdodane via flic.kr/p/o8vboG, CC BY-NC licence
- A380 (slide 34): Antony Stanley via flic.kr/p/8C18fG, CC BY-SA licence
- You talkin’ to me? (slide 36): Robert De Niro in Taxi Driver, 1976
Further reading


- The Threat observatory of Cambridge University’s *Center for risk studies*, cambridgeriskframework.com/taxonomy


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