Economic viewpoint on risk transfer

Eric Marsden
<eric.marsden@risk-engineering.org>

How much risk should my organization take up?
Learning objectives

1. Understand different methods of transferring the financial component of risk
2. Understand concepts of expected value, expected utility and risk aversion
3. Know how to calculate the value of insurance (risk premium)
Which do you prefer?

Option A
1000 € for sure

Option B
50% chance of winning 3000 €
50% chance of winning 0 €
Which do you prefer?

Option A

1000 € for sure

\[ \mathbb{E}(A) = 1000 \, \text{€} \]

Option B

50% chance of winning 3000 €
50% chance of winning 0 €

\[ \mathbb{E}(B) = \frac{1}{2} \times 3000 \, \text{€} + \frac{1}{2} \times 0 = 1500 \, \text{€} \]
Which do you prefer?

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When comparing two gambles, a reasonable start is to compare their expected value.
Expected value

- Expected value of a gamble: the value of each possible outcome times the probability of that outcome

\[ \mathbb{E}(\text{situation}) = \sum_{\text{outcomes } i} \Pr(i) \times W(i) \]

- Interpretation: the amount that I would **earn on average** if the gamble were repeated many times
  - if all probabilities are equal, it’s the *average* value

- For a binary choice between A and B:

\[ \mathbb{E}(W) = \Pr(A) \times W_A + (1 - \Pr(A)) \times W_B \]
Playing black 13 in roulette

The expected value of betting 1€ on black 13 in American roulette (which has 38 pockets numbered 1 to 36 plus 0 plus 00, and a payout for a single winning number of 35 to one) is

$$35 \times \frac{1}{38} + -1 \times \frac{37}{38} = -0.0526 \, €$$

→ Each time you place a bet in the roulette table, you should expect to lose 5.26% of your bet

Note: initial bet is returned as well as 35€ for each euro bet
Finance: Risk as standard deviation of expected value

- Risk in finance (portfolio risk): anticipated variability of the value of my portfolio

- Standard deviation of the expected value of the return on my portfolio
  - return on an investment = next value - present value

- In general, riskier assets have a higher return

- A portfolio manager can reduce risk by diversifying assets
Diversification: example

- Diversification = reducing risk by allocating resources to different activities whose outcomes are not closely related

- Example: company selling air conditioners and heaters

- Assume equiprobability of hot and cold weather

Expected profit as a function of weather and type of equipment sold

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Diversification: example

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Example: company selling air conditioners and heaters

Assume equiprobability of hot and cold weather

If company sells only AC
- $\mathbb{E}(\text{profit}) = 21\,\text{k€}$
- $\sigma(\text{profit}) = 9\,\text{k€}$

If company sells only heaters
- $\mathbb{E}(\text{profit}) = 21\,\text{k€}$
- $\sigma(\text{profit}) = 9\,\text{k€}$

If company sells both
- $\mathbb{E}(\text{profit}) = 21\,\text{k€}$
- $\sigma(\text{profit}) = 0\,\text{€}$

Conclusion: company should sell both to reduce risk

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*Expected profit as a function of weather and type of equipment sold*
The Saint Petersburg game

- You flip a coin repeatedly until a tail first appears
  - the pot starts at 1€ and doubles every time a head appears
  - you win whatever is in the pot the first time you throw tails and the game ends

- For example:
  - T (tail on the first toss): win 1€
  - H T (tail on the second toss): win 2€
  - H H T: win 4€
  - H H H T: win 8€

- Which would you prefer?
  A  10€ for sure
  B  the right to play the St. Petersburg game
The Saint Petersburg game

▷ What is the expected value of the St. Petersburg game?

The probability of throwing a tail on a given round:
• 1st round: \( \Pr(Tails) = \frac{1}{2} \)
• 2nd round: \( \Pr(Heads) \times \Pr(Tails) = \frac{1}{4} \)
• 3rd round: \( \Pr(Heads) \times \Pr(Heads) \times \Pr(Tails) = \frac{1}{8} \)
• \( k \)th round: \( \frac{1}{2^k} \)

▷ How much can you expect to win on average?
• with probability ½ you win 1€, ¼ you win 2€, \( \frac{1}{8} \) you win 4€, \( \frac{1}{16} \) you win 8€ …
• \( \mathbb{E}(\text{win}) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots = \infty \)
The Saint Petersburg game

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The probability of throwing a tail on a given round:

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- \( k^{th} \) round: \( \frac{1}{2^k} \)

How much can you expect to win on average?

- with probability \( \frac{1}{2} \) you win 1€, \( \frac{1}{4} \) you win 2€, \( \frac{1}{8} \) you win 4€, \( \frac{1}{16} \) you win 8€ ...
- \( \mathbb{E} \text{(win)} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + ... = \infty \)
The Saint Petersburg game

▷ Expected value of the game is infinite, and yet few people would be willing to pay more than 20€ to play
  • “the St. Petersburg Paradox”

▷ Bernoulli (1738):
  • the “value” of a gamble is not its monetary value
  • people attach some subjective value, or utility, to monetary outcomes

▷ Bernoulli’s suggestion: people do not seek to maximize expected values, but instead maximize expected utility
  • marginal utility declines as wealth increases (poor people value increments in wealth more than rich people do)
  • an individual is not necessarily twice as happy getting 200€ compared to 100€
  • people are “risk averse”
Utility in classical microeconomics

- Utility: measure of goal attainment or want satisfaction
  - $U(x) =$ utility function for the good $x$

- Utility functions are monotonically increasing: more is preferred to less
  - $U'(x) > 0$

- Marginal utility of $x$: the change in utility resulting from a 1 unit change in $x$
  - $MU(x) \overset{\text{def}}{=} \Delta U(x) / \Delta x$

- Principle of diminishing marginal utility
  - each successive unit of a good yields less utility than the one before it
Expected utility

- Expected *value* is the probability weighted average of the monetary value.
- Expected *utility* is the probability weighted average of the utility from the potential monetary values.

\[
E(U) = \sum_{\text{outcomes}} \Pr(\text{outcome}_i) \times U(\text{outcome}_i)
\]

- *U* is the person’s **von Neumann-Morgenstern utility function**
Terminology: risk and uncertainty

Risk

Future state is unknown.
Probability of each possibility is well-known.

Uncertainty

Possible future states are known.
Probability of each possibility is not well-known.

Radical uncertainty

Future states are not well known or delimited.

Terminology developed in economics, following the work of F. Knight [1923]
Expected utility hypothesis

- People’s preferences can be represented by a function $U$
  - where $U(A) > U(B)$ iff $A > B$ ($A$ is preferred to $B$)

- $U$ is a way of modeling people’s behaviour when faced with risk

The expected utility framework is useful for reasoning about behaviour in situations of risk, but is not a full explanation. The economist Maurice Allais showed that one of the axioms of EU, independence (two gambles mixed with a third one maintain the same preference order as when the two are presented independently of the third one), does not model real behaviour. **Prospect theory** is a more recent theory which models a wider range of real behaviour.
Risk aversion

Reluctance of a person to accept a gamble with an uncertain payoff rather than another gamble with a more certain, but possibly lower, expected payoff.

▷ I have 10€. Suppose I can play a gamble with 50% chance of winning 5€, and 50% chance of losing 5€.

▷ If I refuse to play:
  • Expected value of wealth =
  • Expected utility =

▷ If I play:
  • Expected value of wealth =
  • Expected utility =
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Risk aversion (psychology & economics)

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▷ If I play:
  • Expected value of wealth = 10€
  • Expected utility = $0.5U(15€) + 0.5U(5€)$
Risk aversion and utility function

Play: \( EU = 0.5U(5\€) + 0.5U(15\€) \)
Risk aversion and utility function

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Risk aversion and utility function

Don’t play: $EU = U(10\text{€})$

Typical utility function for risk-averse people: log utility.
If I am risk averse, the utility of gambling is lower than the utility of the sure thing: my utility function is concave.

Typical utility function for risk averse person: log
Attitudes to risk

- Risk attitudes and fair gambles:
  - A **risk averse** person will never accept a fair gamble
  - A **risk loving** person will always accept a fair gamble
  - A **risk neutral** person will be indifferent towards a fair gamble

- Given the choice between earning the same amount of money through a gamble or through certainty,
  - the risk averse person will opt for certainty
  - the risk loving person will opt for the gamble
  - the risk neutral person will be indifferent

- Note: in reality, individual risk attitudes will depend on the context, on the type of risk, etc.
Certainty equivalent value

- The **certainty equivalent value** is the sum of money for which an individual would be indifferent between receiving that sum and taking the gamble.

- The certainty equivalent value of a gamble is less than the expected value of a gamble for risk averse consumers.

- The **risk premium** is the difference between the expected payoff and the certainty equivalent.
  
  - this is the “cost of risk”: the amount of money an individual would be willing to pay to avoid risk
  
  - risk premium = value of insurance
Risk aversion and insurance

- Going without insurance generally has a higher expected value than going with insurance, but the risk is much greater without insurance
  - in roulette, you take a risk by playing
  - in insurance, you pay a company to take a risk for you

- A risk averse person will pay more than the expected value of a game that lets him or her avoid a risk
  - suppose you face a $\frac{1}{100}$ chance of losing 10k€
  - “actuarially fair” value for insurance (expected value): 100€
  - risk averse: you would pay more than 100€ for an insurance policy that would reimburse you for that 10k€ loss, if it happens
Insurance companies

▷ Suppose there are many people like you, and you’d each be willing to pay 110€ to avoid that risk of losing 10k€
  • you join together to form a mutual insurance company
  • each member pays 110€
  • anyone who is unlucky and loses is reimbursed 10k€
  • the insurance company probably comes out ahead
  • the more participants in your mutual insurance company, the more likely it is that you’ll have money left over for administrative costs and profit

▷ How can an insurance company assume all these risks?
  • isn’t it risk averse, too?

▷ The insurance company can do what an individual can’t
  • play the game many times and benefit from the **law of large numbers**
  • the larger an insurance company is, the better it can do this
Aside: insurance and moral hazard

- Insurance companies generally don’t offer *full* insurance.

- They use mechanisms like a *deductible* to make the insured cover a certain proportion (or fixed threshold) of the loss.
  - Example: you must pay the first 600€ of any damage to your car, and the insurance company pays the remaining damage.

- Avoids “moral hazard”: insurance buyer retains an incentive to exercise care to avoid loss.
Willingness to pay for insurance

▷ Consider a person with a current wealth of 100 k€ who faces a 25% chance of losing her automobile, which is worth 20 k€
  • assume that her utility function is \( U(x) = \log(x) \)

▷ The person’s expected utility

\[
\mathbb{E}(U) = 0.75U(100k) + 0.25U(80k) \\
= 0.75\log(100k) + 0.25\log(80k) \\
= 11.45
\]

▷ The individual will likely be willing to pay more than 5 k€ to avoid the gamble. How much will she pay for insurance?

\[
\mathbb{E}(U) = U(100k - y) = \log(100k - y) = 11.45714 \\
100k - y = e^{11.45714} \\
y = 5426
\]

▷ The maximum she is willing to pay is 5426 €
  • her risk premium (the insurance company’s expected profit) = 426 €
Further reading

▷ *Quantum Microeconomics* is an opensource online textbook on introductory and intermediate microeconomics

▷ *Introduction to Economic Analysis* is an opensource textbook on microeconomics

Feedback welcome!

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