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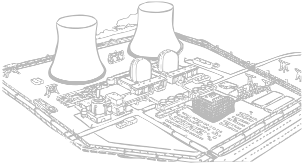

Economic viewpoint on risk transfer

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How much risk should my organization take up?

Learning objectives

1. Understand different methods for transferring the financial component of risk
2. Understand concepts of *expected value*, *expected utility* and *risk aversion*
3. Know how to calculate the value of insurance (risk premium)

Which do you prefer?

Option A

1000 € for sure

Option B

50% chance of winning 3000 €
50% chance of winning 0€

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$$\mathbb{E}(A) = 1000\,€$$

$$\mathbb{E}(B) = \frac{1}{2} \times 3000\,€ + \frac{1}{2} \times 0 = 1500\,€$$

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When comparing two gambles, a reasonable start is to compare their **expected value**

Expected value

- Expected value of a gamble: the value of each possible outcome times the probability of that outcome

$$\mathbb{E}(situation) = \sum_{outcomes\ i} \Pr(i) \times W(i)$$

- Interpretation: the amount that I would **earn on average** if the gamble were repeated many times
  - if all probabilities are equal, it's the *average* value

- For a binary choice between A and B:

$$\mathbb{E}(W) = \Pr(A) \times W_A + (1 - \Pr(A)) \times W_B$$

wealth if outcome A occurs

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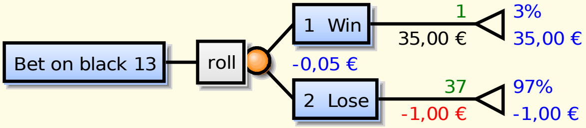
Playing black 13 in roulette

The expected value of betting 1€ on black 13 in American roulette (which has 38 pockets numbered 1 to 36 plus 0 plus 00, and a payout for a single winning number of 35 to one) is

$$35\,€ \times \frac{1}{38} + -1€ \times \frac{37}{38} = -0.0526\,€$$

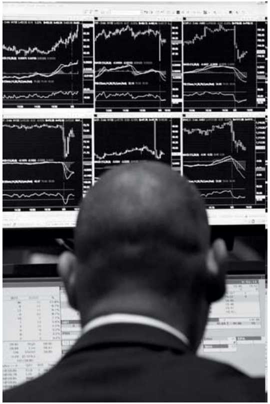
→ Each time you place a bet in the roulette table, you should expect to lose 5.26% of your bet



Note: initial bet is returned as well as 35€ for each euro bet

Finance: risk as standard deviation of expected value

- Risk in finance (portfolio risk): anticipated variability of the value of my portfolio
- Standard deviation of the expected value of the return on my portfolio
  - return on an investment = next value - present value
- In general, riskier assets have a higher return
- A portfolio manager can reduce risk by **diversifying** assets



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Diversification: example

- Diversification = reducing risk by allocating resources to different activities whose outcomes are not closely related
- Example: company selling air conditioners and heaters
- Assume equiprobability of hot and cold weather

Weather	Hot	Cold
AC	30 k€	12 k€
Heaters	12 k€	30 k€

Expected profit as a function of weather and type of equipment sold

Diversification: example

- Diversification = reducing risk by allocating resources to different activities whose outcomes are not closely related
- Example: company selling air conditioners and heaters
- Assume equiprobability of hot and cold weather
- If company sells only AC
  - $\mathbb{E}(\text{profit}) = 21$ k€
  - $\sigma(\text{profit}) = 9$ k€
- If company sells only heaters
  - $\mathbb{E}(\text{profit}) = 21$ k€
  - $\sigma(\text{profit}) = 9$ k€
- If company sells both
  - $\mathbb{E}(\text{profit}) = 21$ k€
  - $\sigma(\text{profit}) = 0$€
- **Conclusion**: company should sell both to reduce risk

Weather	Hot	Cold
AC	30 k€	12 k€
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Expected profit as a function of weather and type of equipment sold

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The Saint Petersberg game

- You flip a coin repeatedly until a tail first appears
  - the pot starts at 1€ and doubles every time a head appears
  - you win whatever is in the pot the first time you throw tails and the game ends
- For example:
  - T (tail on the first toss): win 1€
  - H T (tail on the second toss): win 2€
  - H H T: win 4€
  - H H H T: win 8€
- Which would you prefer?

  A 10€ for sure

  B the right to play the St. Petersburg game

The Saint Petersberg game

- What is the expected value of the St. Petersburg game?

The Saint Petersberg game

- What is the expected value of the St. Petersburg game?
- The probability of throwing a tail on a given round:
  - 1st round: $\Pr(Tails) = \frac{1}{2}$
  - 2nd round: $\Pr(Heads) \times \Pr(Tails) = \frac{1}{4}$
  - 3rd round: $\Pr(Heads) \times \Pr(Heads) \times \Pr(Tails) = \frac{1}{8}$
  - k^{th} round: $\frac{1}{2k}$
- How much can you expect to win on average?
  - with probability ½ you win 1€, ¼ you win 2€, ⅛ you win 4€, 1⁄16 you win 8€ …
  - $\mathbb{E}(win) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \ldots = \infty$

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The Saint Petersberg game

- ▷ Expected value of the game is infinite, and yet few people would be willing to pay more than 20€ to play
  - "the St. Petersburg Paradox"
- ▷ Bernoulli (1738):
  - the "value" of a gamble is not its monetary value
  - people attach some subjective value, or *utility*, to monetary outcomes
- ▷ Bernoulli's suggestion: people do not seek to maximize expected values, but instead maximize *expected utility*
  - marginal utility declines as wealth increases (poor people value increments in wealth more than rich people do)
  - an individual is not necessarily twice as happy getting 200€ compared to 100€
  - people are "risk averse"

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Utility in classical microeconomics

- ▷ Utility: measure of goal attainment or want satisfaction
  - $U(x)$ = utility function for the good x
- ▷ Utility functions are monotonically increasing: more is preferred to less
  - $U'(x) > 0$
- ▷ Marginal utility of x: the change in utility resulting from a 1 unit change in x
  - $MU(x) \stackrel{\text{def}}{=} \frac{\Delta U(x)}{\Delta x}$
- ▷ Principle of *diminishing marginal utility*
  - each successive unit of a good yields less utility than the one before it

Image source: Banksy

Expected utility

- Expected *value* is the probability weighted average of the monetary value
- Expected *utility* is the probability weighted average of the utility from the potential monetary values
- $\mathbb{E}(U) = \sum_{outcomes} \Pr(outcome_i) \times U(outcome_i)$
- U is the person's **von Neumann-Morgenstern utility function**

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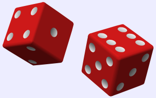
Terminology: risk and uncertainty

Risk

Future state is unknown.

Probability of each possibility is well-known.



Uncertainty

Possible future states are known.

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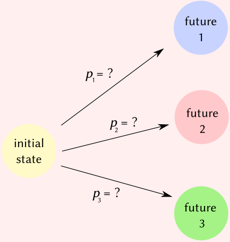
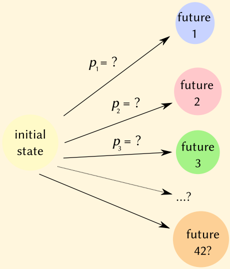


Radical uncertainty

Future states are not well known or delimited.



Terminology developed in economics, following the work of F. Knight [1923]

Expected utility hypothesis

- People's preferences can be represented by a function U
  - where $U(A) > U(B)$ iff $A \succ B$ (A is preferred to B)
- U is a way of modeling people's behaviour when faced with *risk*

The expected utility framework is useful for reasoning about behaviour in situations of risk, but is not a full explanation. The economist Maurice Allais showed that one of the axioms of EU, independence (two gambles mixed with a third one maintain the same preference order as when the two are presented independently of the third one), does not model real behaviour. **Prospect theory** is a more recent theory which models a wider range of real behaviour.

Risk aversion

Risk aversion (psychology & economics)

Reluctance of a person to accept a gamble with an uncertain payoff rather than another gamble with a more certain, but possibly lower, expected payoff.

- ▷ I have 10€. Suppose I can play a gamble with 50% chance of winning 5€, and 50% chance of losing 5€.
- ▷ If I refuse to play:
  - Expected value of wealth =
  - Expected utility =
- ▷ If I play:
  - Expected value of wealth =
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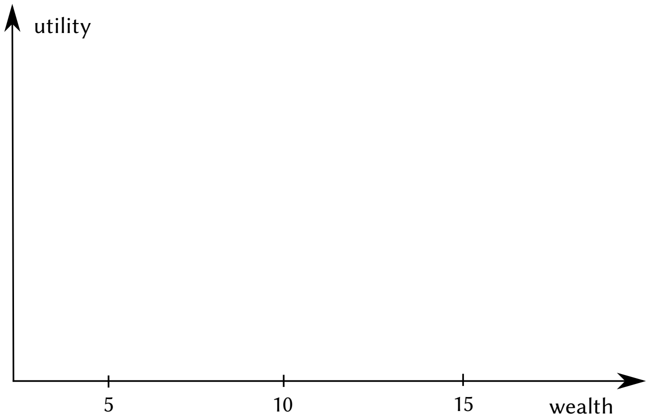
- If I refuse to play:
  - Expected value of wealth = 10€
  - Expected utility = $U(10€)$
- If I play:
  - Expected value of wealth = 10€
  - Expected utility = $0.5U(15€) + 0.5U(5€)$

Risk aversion and utility function

Play: $EU = 0.5U(5€) + 0.5U(15€)$



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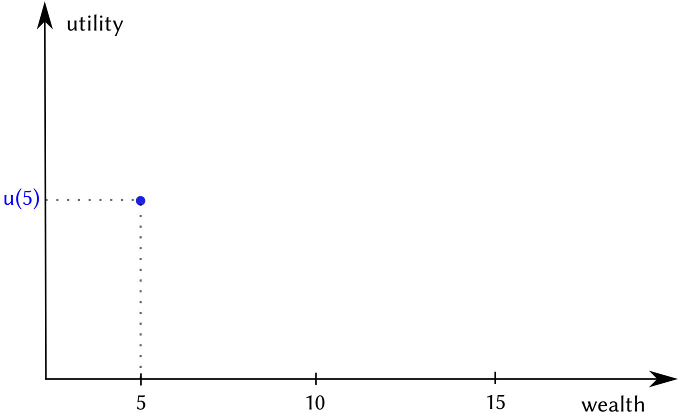

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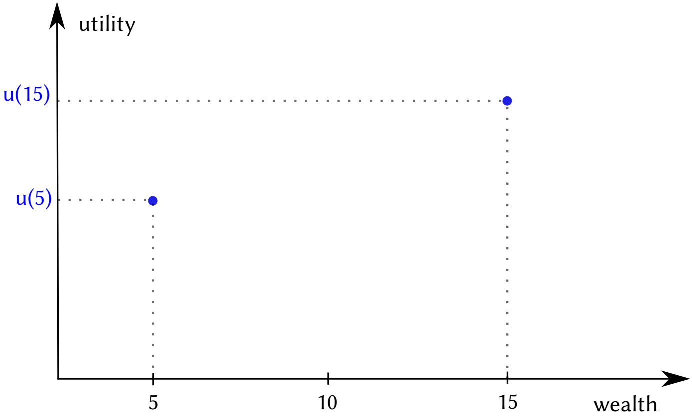

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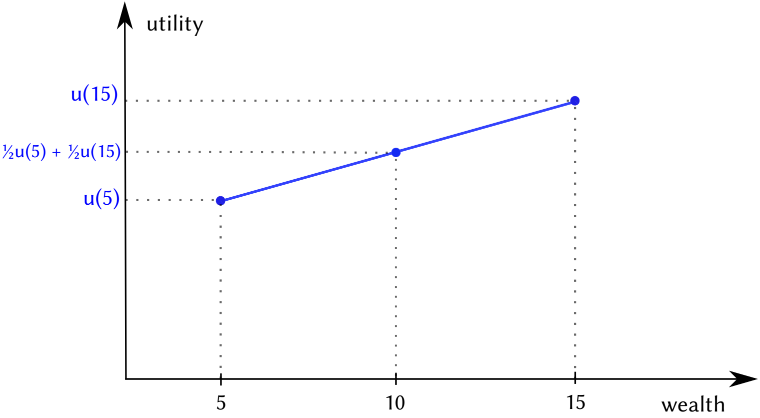
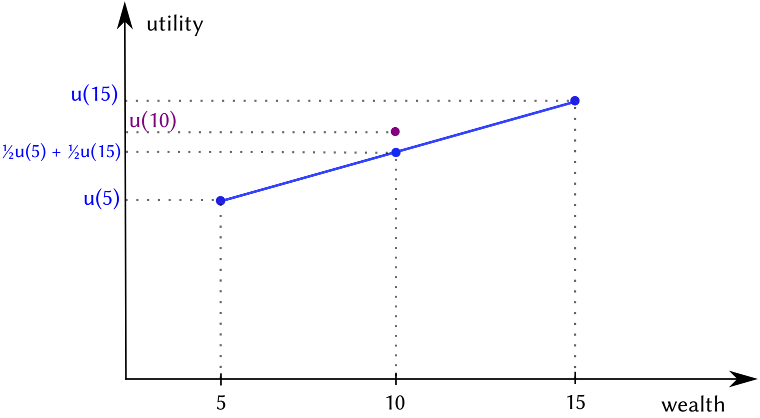


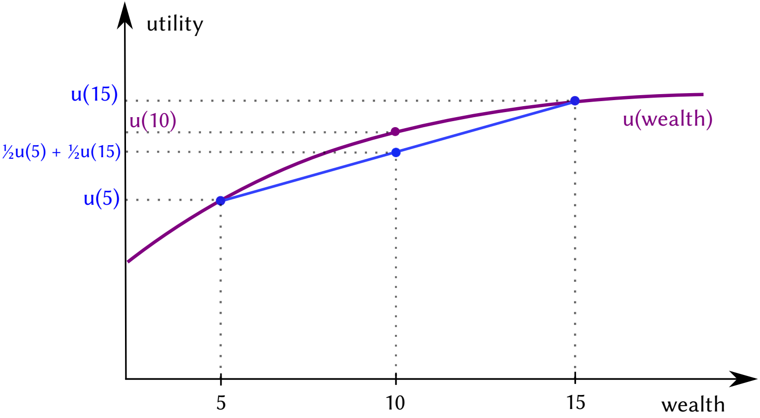
Risk aversion and utility function

Don't play: $EU = U(10€)$



Risk aversion and utility function

If I am risk averse, the utility of gambling is *lower* than the utility of the sure thing: my utility function is *concave*.



Typical utility function for risk averse person: log

Attitudes to risk

- Risk attitudes and fair gambles:
  - A **risk averse** person will never accept a fair gamble
  - A **risk loving** person will always accept a fair gamble
  - A **risk neutral** person will be indifferent towards a fair gamble
- Given the choice between earning the same amount of money through a gamble or through certainty,
  - the risk averse person will opt for certainty
  - the risk loving person will opt for the gamble
  - the risk neutral person will be indifferent
- Note: in reality, individual risk attitudes will depend on the context, on the type of risk, *etc.*

Certainty equivalent value

- The **certainty equivalent value** is the sum of money for which an individual would be indifferent between receiving that sum and taking the gamble
- The certainty equivalent value of a gamble is less than the expected value of a gamble for risk-averse consumers
- The **risk premium** is the difference between the expected payoff and the certainty equivalent
  - this is the "cost of risk": the amount of money an individual would be willing to pay to avoid risk
  - risk premium = value of insurance

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Risk aversion and insurance

- Going without insurance generally has a higher expected value than going with insurance, but the risk is much greater without insurance
  - in roulette, you take a risk by playing
  - in insurance, you pay a company to take a risk for you
- A risk averse person will pay more than the expected value of a game that lets him or her avoid a risk
  - suppose you face a $\frac{1}{100}$ chance of losing 10 k€
  - "actuarially fair" value for insurance (expected value): 100€
  - risk averse: you would pay more than 100€ for an insurance policy that would reimburse you for that 10 k€ loss, if it happens

Insurance companies

- ▷ Suppose there are many people like you, and you'd each be willing to pay 110 € to avoid that risk of losing 10 k€
  - you join together to form a mutual insurance company
  - each member pays 110 €
  - anyone who is unlucky and loses is reimbursed 10 k€
  - the insurance company probably comes out ahead
  - the more participants in your mutual insurance company, the more likely it is that you'll have money left over for administrative costs and profit
- ▷ How can an insurance company assume all these risks?
  - isn't it risk averse, too?
- ▷ The insurance company can do what an individual can't
  - play the game many times and benefit from the **law of large numbers**
  - the larger an insurance company is, the better it can do this

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Aside: insurance and moral hazard

- Insurance companies generally don't offer *full* insurance
- They use mechanisms like a *deductible* to make the insured cover a certain proportion (or fixed threshold) of the loss
  - Example: you must pay the first 600€ of any damage to your car, and the insurance company pays the remaining damage
- Avoids "moral hazard": insurance buyer retains an incentive to exercise care to avoid loss

Willingness to pay for insurance

- Consider a person with a current wealth of 100 k€ who faces a 25% chance of losing her automobile, which is worth 20 k€
  - assume that her utility function is $U(x) = log(x)$
- The person's expected utility

$$\begin{aligned}\mathbb{E}(U) &= 0.75U(100k) + 0.25U(80k)\\ &= 0.75log(100k) + 0.25log(80k)\\ &= 11.45\end{aligned}$$

- The individual will likely be willing to pay more than 5 k€ to avoid the gamble. How much will she pay for insurance?

$$\mathbb{E}(U) = U(100k - y) = log(100k - y) = 11.45714$$

$$100k - y = e^{11.45714}$$

$$y = 5426$$

- The maximum she is willing to pay is 5426 €
  - her **risk premium** (the insurance company's expected profit) = 426 €

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Further reading

- *Quantum Microeconomics* is an opensource online textbook on introductory and intermediate microeconomics
- *Introduction to Economic Analysis* is an opensource textbook on microeconomics
- The report *Risk attitude & economics* introduces standard and behavioral economic theories of risk and uncertainty to non-economists. Freely available from `foncsi.org/en/publications/collections/viewpoints/risk-attitude-economics`

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